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**ELECTROACOUSTICAL IMAGING TECHNIQUE FOR ENCODING  
INCOHERENT RADIANCE FIELDS AS GABOR ELEMENTARY SIGNALS**

**C. L. FALES AND F. O. HUCK**

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**Langley Research Center**  
Hampton, Virginia 23665



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C. L. Fales and F. O. Huck  
NASA Langley Research Center  
Hampton, Virginia 23665

Summary

A technique is presented for directly encoding incoherent radiance fields as Gabor elementary signals. This technique uses an electro-acoustic sensor to modulate the electronic charges induced by the incident radiance field with the electric fields generated by Gaussian modulated sinusoidal acoustic waves. The resultant signal carries the amplitude and phase information required for localizing spatial frequencies of the radiance field. These localized spatial frequency representations provide a link between the either geometric or Fourier transform representations currently used in computer vision and pattern recognition.

## 1. Introduction

Gabor<sup>1,2</sup> introduced time-frequency diagrams to analyze communication systems. These diagrams represent the realizable tradeoff involved in localizing the frequency of a signal: increasing the time duration to improve the accuracy of a frequency measurement would also reduce the accuracy of the time interval determination. To optimize this basic tradeoff, he used Heisenberg's uncertainty principle to postulate a set of elementary signals which have the property that they are maximally localized in time and frequency.

In treating an analogous problem concerned with the thermodynamics of quantum mechanical systems, Wigner<sup>3</sup> introduced a "probability" function of the simultaneous values of the coordinates and momenta. This function is usually called "Wigner distribution function" (WDF) or, in radar processing, ambiguity function. When applied to communication systems, the WDF represents, like the Gabor diagrams, information in terms of localized spatial frequency spectrums. Bastiaans<sup>4,5</sup> first introduced this function into optics to establish a link between geometric and Fourier analyses of optical systems.

Bartelt et al<sup>6</sup> developed optical methods for measuring and displaying time-frequency representations of one-dimensional signals, and used these methods to investigate sound patterns. To introduce their approach, they cited the musical score, which records log frequency versus time, as a familiar example of time-frequency representations. Bamler and Glunder<sup>7,8</sup> first produced localized spatial frequency representations of two-dimensional targets, using coherent optical processing of photographic film transparencies.

We demonstrate analytically that it is also possible to directly encode incoherent radiance fields as elementary signals with an electro-acoustic sensor. The basic approach is to modulate the electronic charges induced by the incident radiance field with the electric fields generated by Gaussian modulated sinusoidal acoustic waves. Our analysis shows that the amplitude and phase information required for constructing a spatial location-spatial frequency representation of the radiance field can be extracted from the sum frequency term of the resultant signal.

It seems reasonable to speculate that spatial location-spatial frequency representations of scenes may become a useful tool for computer vision. For example, Marcelja<sup>9</sup> and Sakitt and Barlow<sup>10</sup> recently have proposed models, based upon Gabor's elementary signals, for the economical encoding of visual information in the cerebral cortex as depicted in Fig. 1. At present, computer vision tends to rely mostly on generic models (such as geometric shapes and allowable relationships between objects) to determine scene characteristics and recognize spatial patterns. The characterization and recognition of targets also is often performed in the Fourier transform domain, using coherent optical processing. The acquisition and reconstruction of elementary signals could establish a link between these two approaches to pattern recognition.

## 2. Gabor Representation

In the Gabor representation, the (one-dimensional) function  $F(t)$  is expanded in terms of the elementary signals

$$g(t; t_m, f_n) = g_s(t; t_m, f_n) + i g_a(t; t_m, f_n), \quad (1)$$

where  $g_s(t)$  and  $g_a(t)$  are the symmetrical and antisymmetrical elements, respectively, given by

$$g_s(t; t_m, f_n) = (2\pi\sigma^2)^{-1/4} e^{-(t-t_m)^2/4\sigma^2} \cos 2\pi f_n(t-t_m) \quad (2a)$$

$$g_a(t; t_m, f_n) = (2\pi\sigma^2)^{-1/4} e^{-(t-t_m)^2/4\sigma^2} \sin 2\pi f_n(t-t_m) \quad (2b)$$

and illustrated in Fig. 2. The signals are centered at the time  $t = t_m$  and the frequency  $f = f_n$ .

For a signal  $\psi(t)$  with Fourier transform  $\hat{\psi}(f)$ , one can define the effective spread in time as

$$\Delta t = [2\pi(\overline{t^2} - \bar{t}^2)]^{1/2}, \quad (3a)$$

and the effective spread in frequency as

$$\Delta f = [2\pi(\overline{f^2} - \bar{f}^2)]^{1/2} \quad (3b)$$

where the bar indicates an average weighted by  $|\psi(t)|^2$  or  $|\hat{\psi}(f)|^2$ . The Gabor elementary signals then satisfy  $\Delta t \Delta f = 1/2$ , while for any other function  $\Delta t \Delta f > 1/2$ .

Thus, the function  $F(t)$  can be represented as a sum of elementary signals

$$F(t) = \sum_m \sum_n G_{mn} g(t; t_m, f_n). \quad (4)$$

The locations  $t_m$  and  $f_n$  are usually, but not necessarily, regularly spaced, e.g.  $t_m = m \Delta t$ ,  $f_n = \frac{n}{\Delta t}$ . So long as the sampling density conforms to the sampling theorem, i.e., one (two) degree(s) of freedom per cell size  $\Delta t \Delta f = \text{one-half (one)}$ , the rectangular cell shapes are arbitrary.

Equation (4) can be equivalently written for a (two-dimensional) spatial location-spatial frequency domain as

$$F(x,y) = \sum_m \sum_n G_{mn} g(x,y; X_m, Y_m; \nu_n, \omega_n), \quad (5)$$

where  $(X_m, Y_m)$  represents the spatial location and  $(\nu_n, \omega_n)$  the spatial frequency of the elementary cell, and

$$g(x,y; X_m, Y_m, \nu_n, \omega_n) = [2\pi\sigma_x(m)\sigma_y(m)]^{-1/2} e^{-(x-x_m)^2/4\sigma_x^2(m) - (y-y_m)^2/4\sigma_y^2(m) - i2\pi\nu_n(x-x_m) - i2\pi\omega_n(y-y_m)}$$

For the most straightforward generation of Gabor complex spectrums with the direct electronic Fourier transform (DEFT) device,<sup>11,12</sup> we assume all elemental cells are of uniform shape, letting  $\sigma_x(m) = \sigma_x$  and  $\sigma_y(m) = \sigma_y$ . The elementary cells sufficient to satisfy the generalized sampling theorem and, hence, represent an arbitrary function, are  $\Delta x \Delta \nu = \Delta y \Delta \omega = 1$  where we define  $\Delta x = 2\sqrt{\pi}\sigma_x$  and  $\Delta y = 2\sqrt{\pi}\sigma_y$ . The 4-space elemental volume is  $V_4 = \Delta x \Delta y \Delta \nu \Delta \omega = 1$ . Note that Eq. (5) is a complex expansion where there are two degrees of freedom associated with each  $G_{mn}$  and the entire frequency plane is used. For real functions,  $F(x,y)$ , it is only necessary to cover a frequency half-plane.

### 3. Electroacoustic Imaging Device

The current  $I(t)$  generated by an electroacoustic imaging device is proportional to

$$I(t) = \iint dx dy L_s(x,y) E_z^2(x,y,t), \quad (6)$$

where  $E_z(x,y,t) = E_{1z}(x,y,t) + E_{2z}(x,y,t)$  represents the normal component of the electric field associated with the two acoustic traveling waves that modulate the electronic charges induced by the image radiance field  $L_s(x,y)$ , as illustrated in Fig. 3. The two orthogonal acoustic waves are generated by the Gaussian modulated sinusoidal signals as suggested by the Gabor representation and given by

$$s_1(t) = v_1 e^{-(t-T_1)^2/4\sigma_1^2} \cos 2\pi f_1(t-\tau_1) \quad (7a)$$

$$s_2(t) = v_2 e^{-(t-T_2)^2/4\sigma_2^2} \cos 2\pi f_2(t-\tau_2) \quad (7b)$$

for the  $x$  and  $y$  inputs, respectively. The resulting traveling electric waves are given by

$$E_{1z}(x,y,t) = E_{1z} e^{-(t - \frac{x}{v_1} - T_1)^2/4\sigma_1^2} \cos 2\pi f_1(t - \frac{x}{v_1} - \tau_1) \quad (8a)$$

$$E_{2z}(x,y,t) = E_{2z} e^{-(t - \frac{y}{v_2} - T_2)^2/4\sigma_2^2} \cos 2\pi f_2(t - \frac{y}{v_2} - \tau_2) \quad (8b)$$

where  $v_1, v_2$  are the respective surface acoustic wave velocities.

To insure that the portion of the integrand of Eq. (6) resulting as the coefficient of the desired sensing frequency sinusoid varies slowly with respect to the sensing frequency, the sum frequency ( $f_1 + f_2$ ) term is selected and the difference ( $f_1 - f_2$ ) and double ( $2f_1, 2f_2$ ) frequency terms are discriminated against. The sum frequency component,  $I_+(t)$ , is

$$I_+(t) = R_e[\tilde{I}_+(t) E_{1z} E_{2z} e^{i2\pi(f_1 + f_2)t}]. \quad (9)$$

The phasor signal  $I_+(t)$  can be written explicitly as a function of spatial location  $(x_0, y_0)$  and spatial frequency  $(\nu, \omega)$  as

$$I_+(x_0, y_0; \nu, \omega) = e^{-i(\phi_1 + \phi_2)} \iint dx dy L_s(x, y) e^{\{-(x-x_0)^2/4\sigma_x^2 - (y-y_0)^2/4\sigma_y^2 - i2\pi(\nu x + \omega y)\}} \quad (10)$$

where the peak of the Gaussian window pulse that propagates across the electroacoustic device is located at  $x_0 = v_1(t - T_1)$  and  $y_0 = v_2(t - T_2)$ ,  $\nu = f_1/v_1$ ,  $\omega = f_2/v_2$ ,  $\phi_1 = 2\pi f_1 T_1$ ,  $\phi_2 = 2\pi f_2 T_2$ ,  $\sigma_x = v_1 \sigma_1$ ,  $\sigma_y = v_2 \sigma_2$ .

For devices that operate in the 100 MHz range and SAW velocities  $4 \times 10^5$  cm/sec, the center spatial frequencies are rather high, of the order of  $f/v = 250$  cycles/cm. It is thus necessary to sample the radiance field in order to gain access to the lower spatial frequencies of the complex spectrogram. This is accomplished by an interdigital contact pattern in one direction and etching of the photoconductor layer in the other.<sup>11,12</sup> Neglecting the finiteness of the sampling array, the spatial frequency spectrum of the sampled radiance field is given by

$$L_s(\nu, \omega) = \frac{\delta x \delta y}{XY} \sum_{m,n} \hat{L}\left(\nu - \frac{n}{X}, \omega - \frac{m}{Y}\right) \text{sinc}\left(\frac{n\delta x}{X}\right) \text{sinc}\left(\frac{m\delta y}{Y}\right), \quad (11)$$



where  $(X,Y)$  are the sampling intervals,  $(\delta x, \delta y)$  are the aperture dimensions and  $\hat{L}(v,\omega)$  is the spectrum of the original field. The reduced spatial frequencies are defined by  $v_0 = v - \frac{1}{X}$ ,  $\omega_0 = \omega - \frac{1}{Y}$ . For  $L(x,y)$  band-limited to  $|v| \leq \frac{1}{2X}$ ,  $|\omega| \leq \frac{1}{2Y}$  and  $(v_0, \omega_0)$  restricted to the same range, it can be shown that for  $\tilde{I}_+(x_0, y_0; v_0, \omega_0) \equiv \tilde{I}_+(x_0, y_0; v_0 + \frac{1}{X}, \omega_0 + \frac{1}{Y})$

$$\tilde{I}_+(x_0, y_0; v_0, \omega_0) = e^{-i(\phi_1 + \phi_2)} \iint dx dy L(x,y) e^{-(x-x_0)^2/4\sigma_x^2 - (y-y_0)^2/4\sigma_y^2} e^{-i2\pi(v_0 x + \omega_0 y)} \quad (12)$$

We define the complex Gabor spectrogram by

$$G(x_0, y_0; v_0, \omega_0) = \iint dx dy L(x,y) g(x,y; x_0, y_0, v_0, \omega_0) \quad (13a)$$

Therefore,

$$G(x_0, y_0; v_0, \omega_0) = (2\pi\sigma_x\sigma_y)^{-1/2} e^{i(\phi_1 + \phi_2) + i2\pi(v_0 x_0 + \omega_0 y_0)} \tilde{I}_+(x_0, y_0; v_0, \omega_0). \quad (13b)$$

The phase factors  $(\phi_1 + \phi_2)$  and  $2\pi(v_0 x_0 + \omega_0 y_0)$  are, in principle, known. Thus, a measurement of the magnitude and phase of the signal current,  $\tilde{I}_+$ , yields the complex Gabor spectrogram. The location  $(x_0, y_0)$  of the peak of the intersecting Gaussian waveforms follows the trajectory

$$y_0 = \left( \frac{v_2}{v_1} \right) x_0 + v_2(T_1 - T_2) \quad (14)$$

as illustrated in Fig 3. The Gabor receptive field for a fixed frequency  $(v_0, \omega_0)$  is moved for that frequency across the entire  $x,y$  image plane by

varying the delays  $T_1, T_2$ . The frequency is then stepped to the adjacent spatial frequency cell, and the receptive field is again moved across the  $x, y$  image; and so forth, until the entire spatial-spatial frequency domain has been covered.

The complex spectrogram and the desired coefficients  $G_{mn}$  in Eq. (5) are completely specified by the value of the complex spectrogram on the Gabor lattice.<sup>13</sup> It is sufficient then to sample the signal current along each scan line at time intervals corresponding through the wave velocities to the Gabor lattice. Although there is some overlap of the Gabor functions, the coefficients  $G_{mn}$  can be approximated by these sampled values of signal current, that is

$$G_{mn} \approx (2\pi\sigma_x\sigma_y)^{-1/2} e^{i(\phi_1 + \phi_2) + i2\pi(\nu_n x_m + \omega_n y_m)} \tilde{I}_+(x_m, y_m; \nu_n, \omega_n). \quad (15)$$

It is interesting to observe that the electroacoustic imaging device could be operated as a conventional imaging system (sum frequency mode) or a Fourier transform system (difference frequency mode) simply by adjusting the standard deviation  $\sigma$  of the Gaussian envelope. As  $\sigma \rightarrow 0$ , the elementary signal becomes a delta function and the device functions as an imaging system; and as  $\sigma \rightarrow \infty$ , the elementary signal becomes a sinusoid and the device functions as a direct electronic Fourier transform (DEFT) system. Other types of complex spectrograms can also be obtained. For example, rather than the Gaussian windows, a rectangular window function of size  $\Delta x \Delta y$  may be applied. In this case, the sampled signal current values are exactly the desired expansion coefficients of the associated "Gabor type" expansion.

### Concluding Remarks

It would be difficult to predict the ultimate impact of this approach--that is, of the sensing, characterization, and recognition of elementary signals--on artificial vision.

Clearly, however, one should not be apprehensive because it initially may appear to be rather abstract and perhaps even alien to our intuitive perception of our own visual processes. This approach is, as Gabor's aptly phrased designation "elementary signal" suggests, deeply rooted in mathematics concerned with characterizing natural phenomena.

Perhaps it would be more appropriate to regard this approach merely as another transform for encoding signals. As such, it might be expected to have certain advantages for encoding spatial features and patterns for subsequent autonomous characterization and recognition. Our expectation arises primarily from two observations: one, some recent efforts to model human visual processes are based on Gabor's concept of elementary signals as the basic visual information-carrying signals from eye to brain: and, two, a recent laboratory realization of elementary signal representation of spoken words indicates that this type of representation is particularly suitable for generating recognizable speech patterns.

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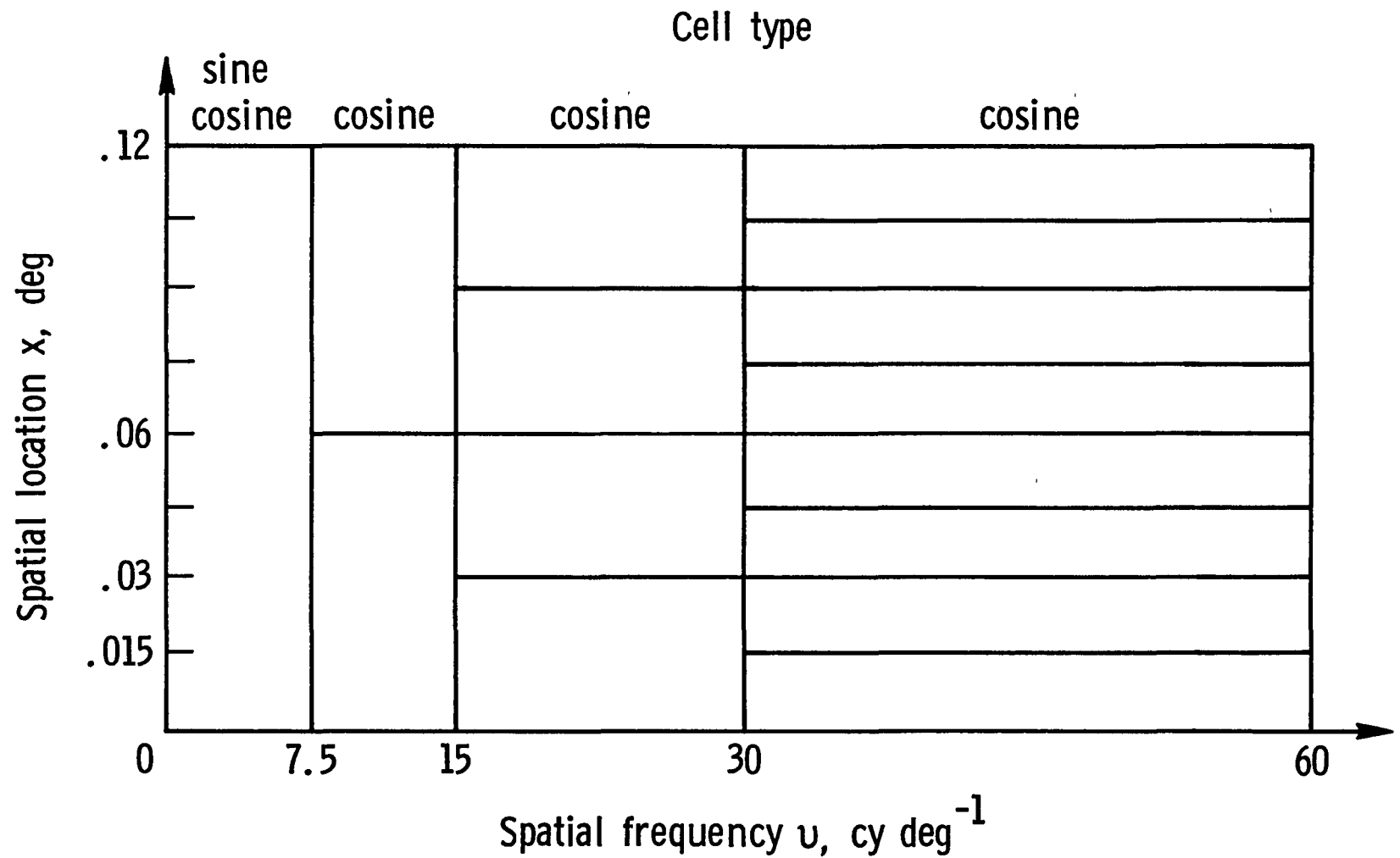


FIG. 1. GABOR REPRESENTATION OF A MODEL PROPOSED BY SAKITT AND BARLOW<sup>10</sup> FOR THE FIRST STAGE OF THE CORTICAL TRANSFORMATION OF VISUAL IMAGES. THE ARRANGEMENT OF CELLS FOR ENCODING TWO-DIMENSIONAL IMAGES IS DISCUSSED IN REF. 10.

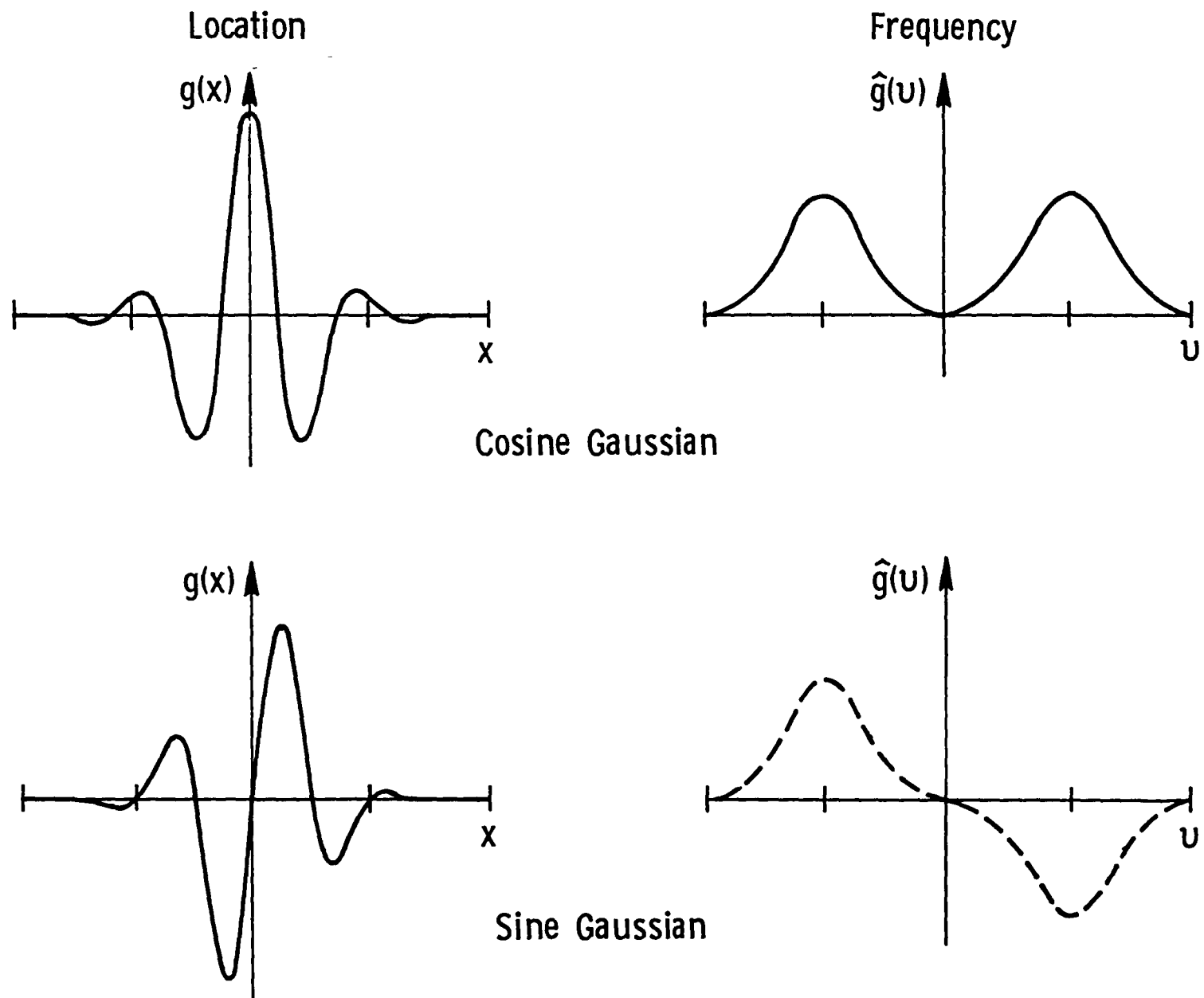
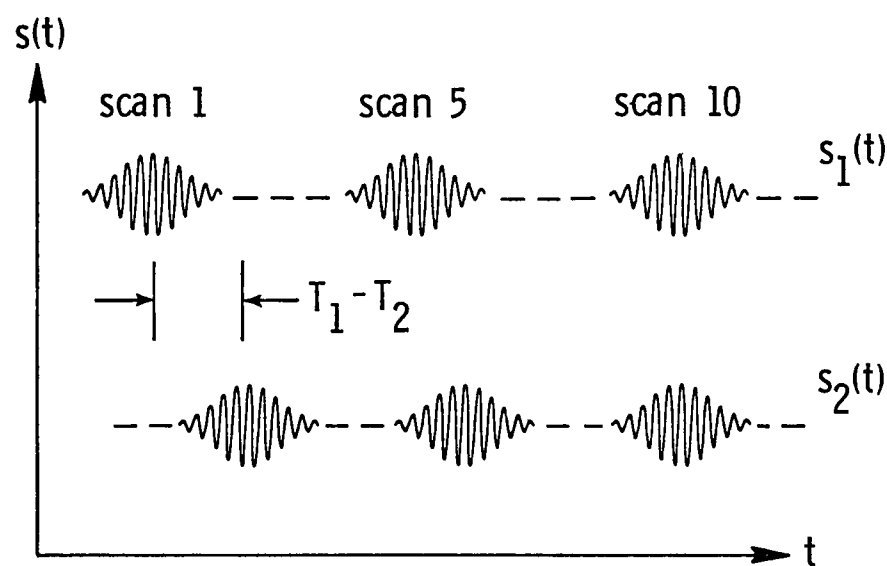
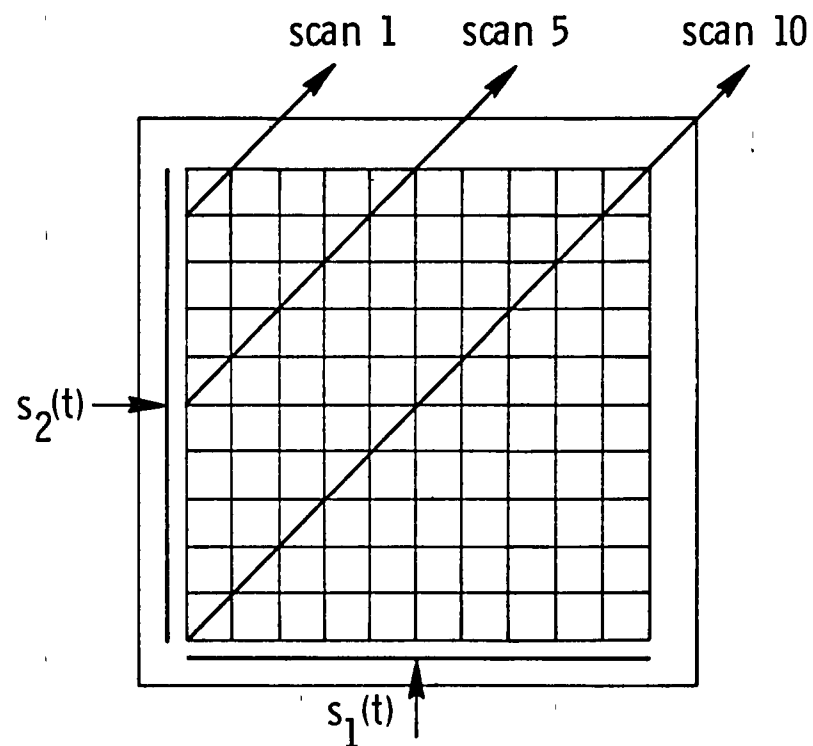


FIG. 2. GABOR FUNCTIONS AND THEIR FOURIER TRANSFORMS.



(a) Gaussian modulated sinusoidal signal.



(b) Scan pattern of Gaussian modulated sinusoidal signal.

FIG. 3. OPERATION OF ELECTROACOUSTIC DEVICE TO OBTAIN ELEMENTARY SIGNAL REPRESENTATIONS OF INCOHERENT RADIANCE FIELDS.



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